Calculus Warm Up #8-5

Expand the log expression

1. $\ln\left(\frac{5x^2}{y}\right)^3$

$3(\ln 5 + 2\ln x - \ln y)$

$3\ln 5 + 6\ln x - 3\ln y$

2. $\ln\frac{\sqrt{3x-5}}{7x^3}$

$\frac{1}{2}\ln(3x-5) -$

$(\ln 7 + 3\ln x)$

Condense the log expression

3. $2\ln(x+2) - \frac{1}{3}(\ln x + \ln y)$

$\ln\frac{(x+2)^2}{\sqrt[3]{xy}}$

4. $\frac{1}{5}\left[3\ln(x+3) + \ln x - \ln(x^2-1)\right]$

$\ln\sqrt[5]{\frac{x(x+3)^3}{x^2-1}}$

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In Exercises 27–30, find the second derivative of the exponential function.

29. $g(x) = (1 + 2x)e^{4x}$

product rule

$$g'(x) = (1+2x)4e^{4x} + 2(e^{4x})$$

$$2e^{4x}\left(\overset{2+4x}{2(1+2x)}+1\right)$$

$$g'(x) = 2e^{4x}(4x+3)$$

$$g''(x) = 2e^{4x}(4) + (4x+3)8e^{4x}$$

$$e^{4x}(8+32x+24)$$

$$g''(x) = e^{4x}(32x+32)$$

$$= 32e^{4x}(x+1)$$

In Exercises 31–34, find the extrema and the points of inflection (if any exist) and sketch the graph of the function.

31. $f(x) = \dfrac{2}{1 + e^{-x}}$ $\Rightarrow 2(1+e^{-x})^{-1}$

$$f'(x) = -2(1+e^{-x})^{-2}(-e^{-x})$$

$$= \frac{2e^{-x}}{(1+e^{-x})^2} \quad (1+e^{-x})^{-2}$$

Always inc.

$$f''(x) = \cancel{2e^{-x}}\boxed{(-2\cancel{(1+e^{-x})}^{-3})}(-e^{-x}) + \boxed{\cancel{(1+e^{-x})}^{-2+1}}(-\cancel{2e^{-x}})$$

$$2e^{-x}(1+e^{-x})^{-3}\left(2e^{-x}-(1+e^{-x})\right)$$

$$0 = \frac{2e^{-x}(1e^{-x}-1)}{(1+e^{-x})^3}$$

$0 \neq 2e^{-x}$

$0 = e^{-x} - 1$

$1 = \dfrac{1}{e^x}$

$e^x = 1$

$x = 0$

$f(0) = \dfrac{2}{1+e^0}$

$= \dfrac{2}{2}$

$= 1$

pt of inflection $(0,1)$

In Exercises 31–34, find the extrema and the points of inflection (if any exist) and sketch the graph of the function.

$f'(x)=0$

$f''(x)=0$

product rule

33. $f(x) = x^2e^{-x}$

$$f'(x) = x^2(-e^{-x}) + 2x(e^{-x})$$

$$= e^{-x}(2x - x^2)$$

$$e^{-x} \neq 0 \qquad x(2-x) = 0$$

$$x = 0 \qquad x = 2$$

35. Find an equation of the line normal to the graph of $y = e^{-x}$ at $(0, 1)$.

7.5 Derivatives of logarithmic functions

To find $\frac{d}{dx}\left(\ln x\right)$

we will use what we know about inverses:

$$f\left(g(x)\right) = x \quad \text{if } f \text{ \& } g \text{ are inverses}$$

So: $$\frac{d}{dx}\left[f\left(g(x)\right)\right] = \frac{d}{dx}\left[x\right]$$

$$f'\left(g(x)\right) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'\left(g(x)\right)}$$

Let $g(x) = \ln x$

$g^{-1}(x) \rightarrow f(x) = e^x$

$f'(x) = e^{(x)}$

Put with our new formula:

$$g'(x) = \frac{1}{f'(g(x))}$$

inverses have reciprocal slopes

the slope of f @ (a, b) is reciprocal to the slope of g @ (b, a)

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{f'(\ln x)}$$

$$= \frac{1}{e^{\ln x}}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

THEOREM 7.9 DERIVATIVE OF THE NATURAL LOGARITHMIC FUNCTION

Let u be a differentiable function of x.

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0 \qquad \frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}, \quad u > 0$$

Notice: Chain rule applies
and can't take the log of a negative!

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

1) $\frac{d}{dx}[\ln(2x)]$ $\quad u = 2x$, $u' = 2$

$$= \frac{\cancel{2}}{\cancel{2}x}$$

$$= \frac{1}{x}$$

2) $\frac{d}{dx}[\ln(x^2 + 1)]$

$$\boxed{= \frac{2x}{x^2+1}}$$

3) $\frac{d}{dx}[x \cdot \ln x]$ product rule

$$= x \cdot \frac{1}{x} + 1(\ln x)$$

$$\boxed{= 1 + \ln x}$$

EXAMPLE 2 Logarithmic properties as an aid to differentiation

Differentiate

$f(x) = \ln \sqrt{x+1}$. $= \ln (x+1)^{\frac{1}{2}}$

$= \frac{1}{2} \ln (x+1)$

$f'(x) = \frac{1}{2} \cdot \frac{1}{x+1}$

$\boxed{f'(x) = \frac{1}{2(x+1)}}$

EXAMPLE 3 Logarithmic properties as an aid to differentiation

~~product rule~~ or properties of logs

Differentiate

$f(x) = \ln [x\sqrt{1-x^2}]$.

$\ln x + \frac{1}{2} \ln (1-x^2)$

$f'(x) = \frac{1}{x} + \frac{1}{2} \left(\frac{-2x}{1-x^2}\right)$

$f'(x) = \frac{1}{x}\left(\frac{1-x^2}{1-x^2}\right) + \frac{-x}{1-x^2}\left(\frac{x}{x}\right)$

$= \frac{1-x^2-x^2}{x(1-x^2)}$

$f'(x) = \frac{1-2x^2}{x(1-x^2)}$

EXAMPLE 4 Logarithmic properties as an aid to differentiation

Differentiate

$$f(x) = \ln \frac{x(x^2+1)^2}{\sqrt{2x^3-1}}.$$

$$f(x) = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$$

$$f'(x) = \frac{1}{x} + 2\left(\frac{2x}{x^2+1}\right) - \frac{1}{\cancel{2}}\left(\frac{\cancel{6}^{3}x^2}{2x^3-1}\right)$$

$$= \frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}$$

lets not put these fractions together!

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